

Mark Scheme (Results)

January 2014

Pearson Edexcel International Advanced Level

Further Pure Mathematics 1 (6667A/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2014
Publications Code IA037751
All the material in this publication is copyright
© Pearson Education Ltd 2014

PMT

General Marking Guidance

 All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PMT

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where $|pq| = |c|$, leading to $x = (ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = (ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = (ax^2 + bx + c) = (ax^2 + bx +$

2. Formula

Attempt to use <u>correct</u> formula (with values for a, b and c).

3. Completing the square

Solving
$$x^2 + bx + c = 0$$
: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = ...$

Method marks for differentiation and integration:

1. <u>Differentiation</u>

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Notes	Ма	rks
1.	$f(x) = 2x - 5\cos x$, x measured in radians			
(a)	f(1) = -0.7015115293	Either any one of $f(1) = awrt - 0.7$ or	N/1	
	f(1.4) = 1.950164285	f(1.4) = 1.9 or awrt 2.0	M1	
	Sign change (and $f(x)$ is continuous) therefore a root α exists between $x = 1$ and $x = 1.4$	both values correct, sign change and conclusion	A1	
				[2]
(b)	$f(1.2) = 0.5882112276 $ $\{ \Rightarrow 1 \le \alpha \le 1.2 \}$	f(1.2) = awrt 0.6	B1	
		Attempt to find $f(1.1)$	M1	
	f(1.1) = -0.06798060713	f(1.1) = -0.06 or awrt -0.07 with		
	$\Rightarrow 1.1 \le \alpha \le 1.2$	$1.1 \le \alpha \le 1.2$ or $1.1 < \alpha < 1.2$	A1	
		or $[1.1, 1.2]$ or $(1.1, 1.2)$.		
	_			[3] 5

Question Number	Scheme	Notes	Marl	ks
2.	$\mathbf{A} = \begin{pmatrix} -4 & 10 \\ -3 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$			
(i)	$\det \mathbf{A} = (-4)(k) - (-3)(10)$	Applies " $ad \pm bc$ " to A	M1	
	$\Rightarrow -4k + 30 = 2$ or $-4k + 30 = -2$	Equates their det A to either 2 or -2	dM1	
	$\Rightarrow k = 7 \text{ or } k = 8$	Either $k = 8$ or $k = 7$	A1	
	$\rightarrow k - 7$ Of $k - 6$	Both $k = 8$ and $k = 7$	A1	
				[4]
(ii)	$\mathbf{B} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix}$			
	(1, 2, 3)(2, 8)	Writes down a complete 2×2 matrix.	M1	
	$\mathbf{BC} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -3 & -8 \end{pmatrix}$	Any 3 out of 4 elements correct	A1	
	$\begin{pmatrix} -2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \end{pmatrix} \begin{pmatrix} -3 & -8 \end{pmatrix}$	Correct answer.	A1	
				[3] 7

Question Number	Scheme	Notes	Marks
3.	$x = 2t, \ y = \frac{2}{t}, \ t \neq 0$		
	$t = \frac{1}{2} \Rightarrow P(1, 4), t = 4 \Rightarrow Q\left(8, \frac{1}{2}\right)$	Coordinates for either P or Q are correctly stated. (Can be implied).	B1
	$m(PQ) = \frac{\frac{1}{2} - 4}{8 - 1} \left\{ = -\frac{1}{2} \right\}$	An attempt to find the gradient of the chord PQ .	M1
	m(L) = 2	Applying $m(L) = \frac{-1}{\text{their } m(PQ)}$ y = 2x	M1
	So, $L: y = 2x$	y = 2x	A1 oe
			[4] 4

Question Number	Scheme	Notes	Marks
4.	$f(x) = 2\sqrt{x} - \frac{6}{x^2} - 3, x > 0$		
	$f'(x) = x^{-\frac{1}{2}} + 12x^{-3} \left\{ + 0 \right\}$ $f(3.5) = 0.2518614684$ $\left\{ f'(3.5) = 0.8144058657 \right\}$	$\pm \lambda x^{-\frac{1}{2}}$ or $\pm \mu x^{-3}$ Correct differentiation f (3.5) = awrt 0.25	M1 A1 B1
	$\beta = 3.5 - \left(\frac{"0.2518614684"}{"0.8144058657"}\right)$ $= 3.190742075$	Correct application of Newton-Raphson using their values.	M1
	= 3.191 (3dp)	3.191	A1 cao [5] 5

Question Number	Scheme	Notes	Marks
5.	$z = 5 + i\sqrt{3}, w = \sqrt{3} - i$		
(a)	$ w = \left\{ \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} \right\} = 2$	2	B1
(b)	$zw = \left(5 + i\sqrt{3}\right)\left(\sqrt{3} - i\right)$		[1]
	$= 5\sqrt{3} - 5i + 3i + \sqrt{3}$ $= 6\sqrt{3} - 2i$	Either the real or imaginary part is correct. $6\sqrt{3}-2i$	M1 A1 [2]
(c)	$\frac{z}{w} = \frac{\left(5 + i\sqrt{3}\right)}{\left(\sqrt{3} - i\right)} \times \frac{\left(\sqrt{3} + i\right)}{\left(\sqrt{3} + i\right)}$	Multiplies by $\frac{\left(\sqrt{3} + i\right)}{\left(\sqrt{3} + i\right)}$	
	$=\frac{5\sqrt{3}+5i+3i-\sqrt{3}}{3+1}$	Simplifies realising that a real number is needed on the denominator and applies $i^2 = -1$ on their numerator expression and denominator expression.	M1
	$\left\{ = \frac{4\sqrt{3} + 8i}{4} \right\} = \sqrt{3} + 2i$	$\sqrt{3} + 2i$	A1
(d)	$z + \lambda = 5 + i \sqrt{3} + \lambda = (5 + \lambda) + i \sqrt{3}$		[3]
	$\left\{\arg(z+\lambda) = \frac{\pi}{3} \Rightarrow \right\} \frac{\sqrt{3}}{5+\lambda} = \tan\left(\frac{\pi}{3}\right)$	$\frac{\sqrt{3}}{\text{their combined real part}} = \tan\left(\frac{\pi}{3}\right)$	M1 oe
	$\left\{ \frac{\sqrt{3}}{5+\lambda} = \frac{\sqrt{3}}{1} \Rightarrow 5+\lambda = 1 \Rightarrow \right\} \lambda = -4$	-4	A1
			[2] 8

Question Number	Scheme	Notes	Marks
6. (a)	$\sum_{r=1}^{n} r(r+1)(r-1) = \sum_{r=1}^{n} (r^{3} - r)$		
	$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	An attempt to use at least one of the standard formulae correctly.	M1
	$= \frac{1}{4}n(n+1)(n(n+1)-2)$	Correct expression. An attempt to factorise out at least $n(n + 1)$.	A1 M1
	$= \frac{1}{4}n(n+1)(n^2+n-2)$	Achieves the correct answer.	
	$= \frac{1}{4}n(n+1)(n-1)(n+2)$	(Note: $a = 2$).	A1 [4
	$\sum_{r=1}^{n} r(r+1)(r-1) = 10 \sum_{r=1}^{n} r^{2}$		
(b)	$\frac{1}{4}n(n+1)(n-1)(n+2) = \frac{10}{6}n(n+1)(2n+1)$	Sets their part (a) = $\frac{10}{6}n(n+1)(2n+1)$	M1
	$\frac{1}{4}(n-1)(n+2) = \frac{5}{3}(2n+1)$		
	$3(n^2 + n - 2) = 20(2n + 1)$	Manipulates to a " $3TQ = 0$ ".	M1
	$3n^2 - 37n - 26 = 0$	$3n^2 - 37n - 26 = 0$	A1
	(3n+2)(n-13) = 0	A valid method for factorising a 3TQ.	M1
	n = 13	Only one solution of $n = 13$	A1
			[5

Question Number	Scheme	Notes	Maı	rks
7.	$\mathbf{P} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}, \mathbf{M} = \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$	1		
(a)	$\mathbf{P}^{-1} = \frac{1}{4ab}; \begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix}$	$\frac{1}{4ab}$ Two out of four elements correct. Correct matrix.	B1; M1 A1	[3]
	$\mathbf{M} = \mathbf{PQ}$			
(b)	$\Rightarrow \mathbf{P}^{-1}\mathbf{M} = \mathbf{P}^{-1}\mathbf{PQ} \Rightarrow \mathbf{Q} = \mathbf{P}^{-1}\mathbf{M}$ $\mathbf{Q} = \frac{1}{4ab} \begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix} \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$ $= \frac{1}{4ab} \begin{pmatrix} -8ab & 12ab \\ 0 & 4ab \end{pmatrix}$	Multiples their P^{-1} by M	M1	
	$= \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}$	Two out of four elements correct.	A1	
	$-\begin{pmatrix} 0 & 1 \end{pmatrix}$	Correct matrix.	A1	
				[3] 6

Question Number	Scheme	Notes	Marks
8.	$y^2 = 4ax$, at $P(ap^2, 2ap)$.		
(a)	$y = 2\sqrt{a} x^{\frac{1}{2}} \implies \frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{a} x^{-\frac{1}{2}}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k \; x^{-\frac{1}{2}}$	
	or (implicitly) $2y \frac{dy}{dx} = 4a$	or $k y \frac{\mathrm{d}y}{\mathrm{d}x} = c$	M1
	or (chain rule) $\frac{dy}{dx} = 2a \times \frac{1}{2ap}$	or $\frac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$	
	When $x = a p^2$, $\frac{dy}{dx} = \frac{\sqrt{a}}{\sqrt{a p^2}} = \frac{\sqrt{a}}{\sqrt{a p}} = \frac{1}{p}$ or $\frac{dy}{dx} = \frac{4a}{2(2ap)} = \frac{1}{p}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{p}$	A1
	So $m_N = -p$	Applies $m_N = \frac{-1}{their m_T}$	M1
	N : $y - 2ap = -p(x - ap^2)$	Applies $y - 2ap = (\text{their } m_N)(x - ap^2)$	M1
	$\mathbf{N}: \ y - 2ap = -px + ap^3$		
	$\mathbf{N}: \ \ y + px = ap^3 + 2ap$	Correct solution.	A1 cso *
(b)	$(6a, 0) \Rightarrow 0 + p(6a) = ap^3 + 2ap$	Substitutes $x = 6a$, $y = 0$ into N	[5] M1
(-)	$\Rightarrow 4ap = ap^3 \Rightarrow p = 2$	p = 2	A1
	$x = -a, p = 2 \implies y + 2(-a) = a(2)^3 + 2a(2)$	Substitutes $x = -a$ and their p into N	dM1
	$\Rightarrow y = 8a + 4a + 2a = 14a \Rightarrow D(-a, 14a)$	D(-a, 14a)	A1
(c)	When $p = 2$, $x = a(2)^2 = 4a$	Substitutes their <i>p</i> into $x = a p^2$	[4] M1
	Area(XPD) = $\frac{1}{2}(14a)(5a) = 35a^2$	Applies $\frac{1}{2}$ (their 14a)(their "4a" + a)	M1
	2` /` /	$35a^2$	A1
			[3] 12

Question Number	Scheme	Notes	Mark	ĸs
9.	(3-i)z*+2iz=9-i			
	(3-i)(x-iy) + 2i(x+iy) = 9-i	Substituting $z = x + iy$ and $z *= x - iy$ into $(3 - i)z * + 2iz = 9 - i$	M1	
	3x - 3iy - ix - y + 2ix - 2y = 9 - i	Multiplies out $(3-i)(x-iy)$ correctly. This mark can be implied by correct later working.	A1	
	Re part: $3x - y - 2y = 9$	Equating either real or imaginary parts.	M1	
	Im part: $-3y - x + 2x = -1$	One set of correct equations.	A 1	
		Correct equations.	A1	
	3x - 3y = 9			
	x - 3y = -1			
	$2x = 10 \implies x = 5$	Attempt to solve simultaneous equations to find one of x or y .	ddM1	
	$x - 3y = -1 \implies 5 - 3y = -1 \implies y = 2$	Either $x = 5$ or $y = 2$.	A 1	
		Both $x = 5$ and $y = 2$.	A1	
	$\{z = 5 + 2i\}$			[8] 8
				-

Question Number	Scheme	Notes	Marks
10. (i)	$u_{n+1} = 5u_n + 3$, $u_1 = 3$ and $u_n = \frac{3}{4}(5^n - 1)$ $u_1 = 1$; $u_1 = \frac{3}{4}(5^1 - 1) = \frac{3}{4}(4) = 3$	Check that $u_n = \frac{3}{4}(5^n - 1)$	B1
	So u_n is true when $n = 1$. Assume that for $n = k$ that, $u_k = \frac{3}{4}(5^k - 1)$ is true for	yields 3 when $n = 1$.	
	$k \in \mathbb{Z}^+.$ Then $u_{k+1} = 5u_k + 3$		
	$= 5\left(\frac{3}{4}(5^k - 1)\right) + 3$	Substituting $u_k = \frac{3}{4}(5^k - 1)$ into $u_{k+1} = 5u_k + 3$	M1
	$=\frac{3}{4}(5)^{k+1}-\frac{15}{4}+3$	An attempt to multiply out in order to achieve $\pm \lambda(5^{k+1}) \pm \text{constant}$	M1
	$=\frac{3}{4}(5)^{k+1}-\frac{3}{4}$	2	
	$=\frac{3}{4}(5^{k+1}-1)$	$\frac{3}{4}(5^{k+1}-1)$	A1
	Therefore, the general statement, $u_n = \frac{3}{4}(5^n - 1)$ is true when $n = k + 1$. (As u_n is true for $n = 1$,) then u_n is true for all positive integers by mathematical induction	True when $n = k+1$, then by induction the result is true for all positive integers.	A1
			[5]

Question Number	Scheme	Notes	Marks
	$f(n) = 5(5^n) - 4n - 5$ is divisible by 16		
10. (ii)	f (1) = $5(5^1)$ - 4(1) - 5 = 16, {which is divisible by 16}. { : f (n) is divisible by 16 when $n = 1$.}	Shows that $f(1) = 16$	B1
	Assume that for $n = k$,		
	$f(k) = 5(5^k) - 4k - 5$ is divisible by 16 for $k \in \mathbb{Z}^+$.		
	$f(k+1) - f(k) = 5(5^{k+1}) - 4(k+1) - 5 - (5(5^k) - 4k - 5)$	Applies $f(k+1) - f(k)$. Correct expression for $f(k+1) - f(k)$.	M1 A1
	$= 5(5^{k+1}) - 4k - 4 - 5 - 5(5^k) + 4k + 5$		
	$= 25(5^k) - 4k - 4 - 5 - 5(5^k) + 4k + 5$	Achieves an expression in 5^k .	M1
	$=20(5^k)-4$		
	$=4(5(5^k)-4k-5)+16k+20-4$		
	$=4(5(5^k)-4k-5)+16k+16$		
	=4f(k)+16(k+1)		
	$\therefore f(k+1) = 5f(k) + 16(k+1)$	f(k+1) = 5f(k) + 16(k+1)	A1
	$\{ :: f(k+1) = 5f(k) + 16(k+1), \text{ which is divisible by 16 as } $		
	both $5f(k)$ and $16(k+1)$ are both divisible by 16.}		
	If the result is true for $n = k$, then it is now true for		
	n = k + 1. As the result has shown to be true for $n = 1$,	Correct conclusion	A1 cso
	then the result is true for all n .		[6] 11

Appendix

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
- ft denotes "follow through"
- cao denotes "correct answer only"
- oe denotes "or equivalent"

Other Possible Solutions

Question Number	Scheme	Notes	Marks
7.	$\mathbf{P} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}, \mathbf{M} = \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$		
	$\mathbf{M} = \mathbf{PQ}$		
(b)	$ \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix} \begin{pmatrix} q_1 & q_2 \\ q_3 & q_4 \end{pmatrix} $		
Way 2	$\begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix} \begin{pmatrix} q_1 & q_2 \\ q_3 & q_4 \end{pmatrix}$ $-6 = 3q_1 - 2q_3 \qquad 7 = 3q_2 - 2q_4$ $2 = -q_1 + 2q_3 \qquad -1 = -q_2 + 2q_4$ $= \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}$ Writes	down a relevant pair of simultaneous equations. Can be implied by later working. Two out of four elements correct. Correct matrix.	M1 A1 A1 [3]

Question Number	Scheme	Notes	Marks
Aliter	$f(n) = 5(5^n) - 4n - 5 \text{ is divisible by } 16$		
10. (ii)	$f(1) = 5(5^1) - 4(1) - 5 = 16,$	Shows that $f(1) = 16$	B1
Way 2	{which is divisible by 16}. { \therefore f (n) is divisible by 16 when $n = 1$.}		
	Assume that for $n = k$,		
	$f(k) = 5(5^k) - 4k - 5$ is divisible by 16 for $k \in \mathbb{Z}^+$.		
	$f(k+1) = 5(5^{k+1}) - 4(k+1) - 5$	Applies $f(k+1)$.	M1
		Correct expression for $f(k+1)$.	A1
	$=25\left(5^{k}\right)-4k-9$	Achieves an expression in 5^k .	M1
	$=5(5(5^{k})-4k-5)+20k+25-4k-9$		
	$= 5(5(5^{k}) - 4k - 5) + 16(k + 1)$		
	$\therefore f(k+1) = 5f(k) + 16(k+1)$	f(k+1) = 5f(k) + 16(k+1)	A1
	$\{ : f(k+1) = 5f(k) + 16(k+1) \}$, which is divisible by 16		
	as both $5f(k)$ and $16(k+1)$ are both divisible by 16.}		
	If the result is true for $n = k$, then it is now true for		
	n = k+1. As the result has shown to be true for $n = 1$,	Correct conclusion	A1 cso
	then the result is true for all n .		
			[6]

